The 2nd Workshop on Railway Operation for Safety and Reliability

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Optimization of Passenger Railway System Design

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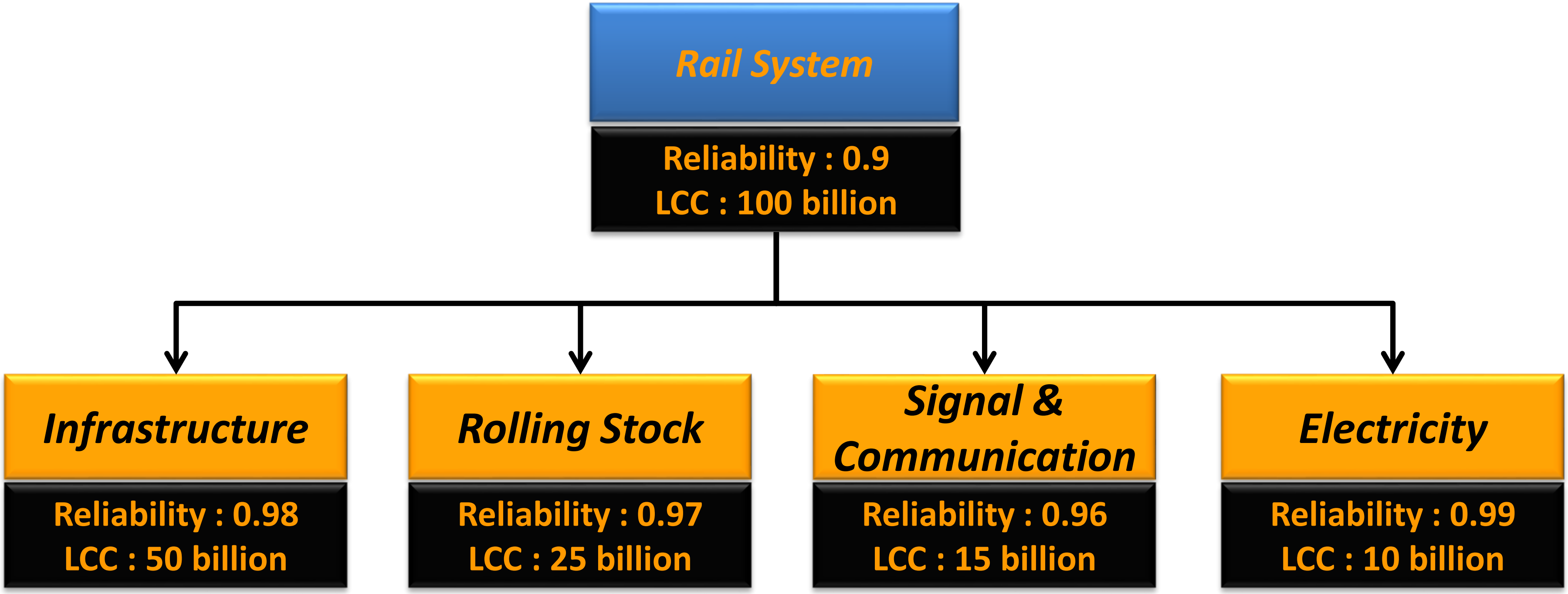
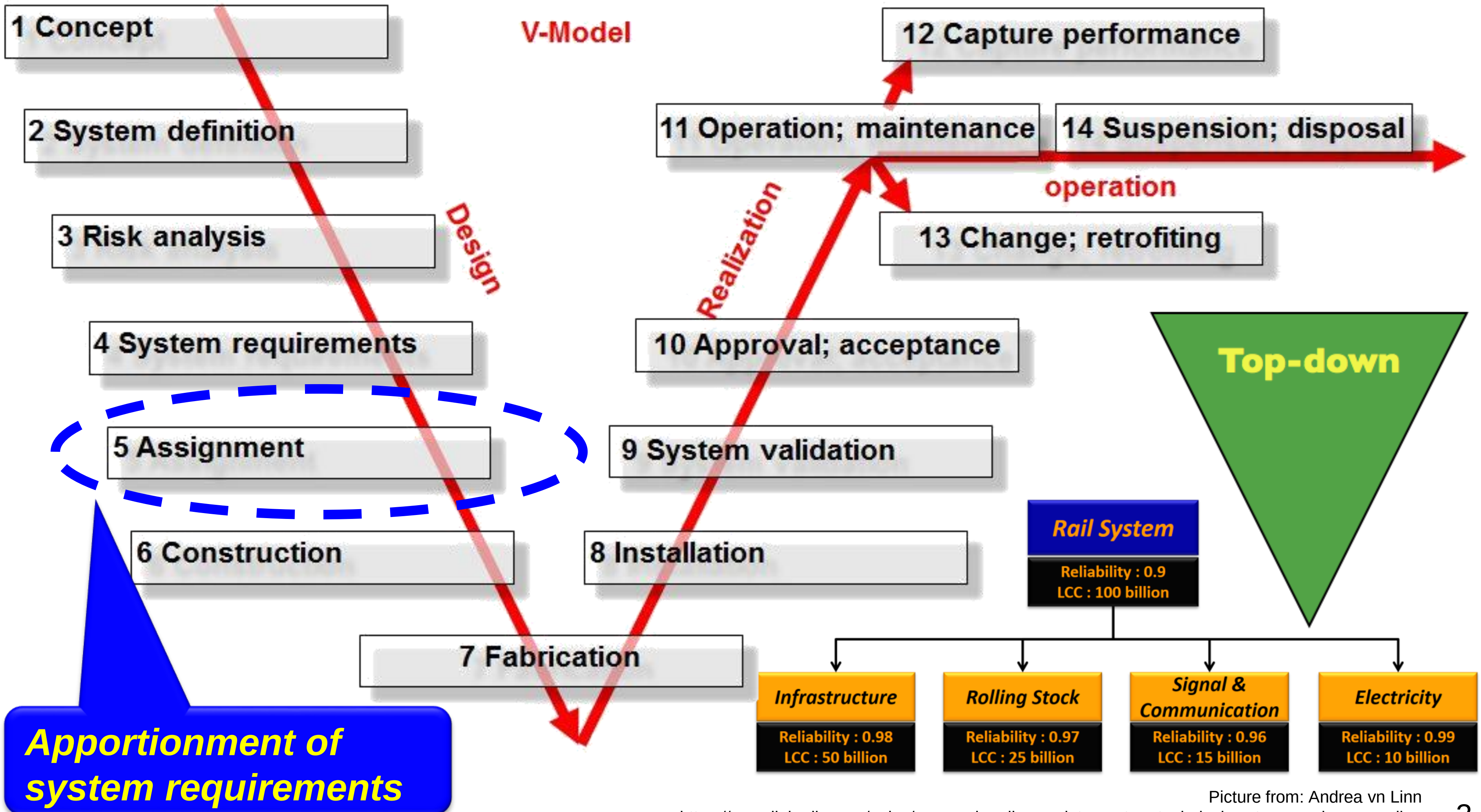
System design considers trade-off between cost and reliability

- A rail system consists of a number of subsystems, and each has its own cost and reliability
- Planners have to carefully allocate the reliability and budget by examining the trade-off between **cost** and **reliability**



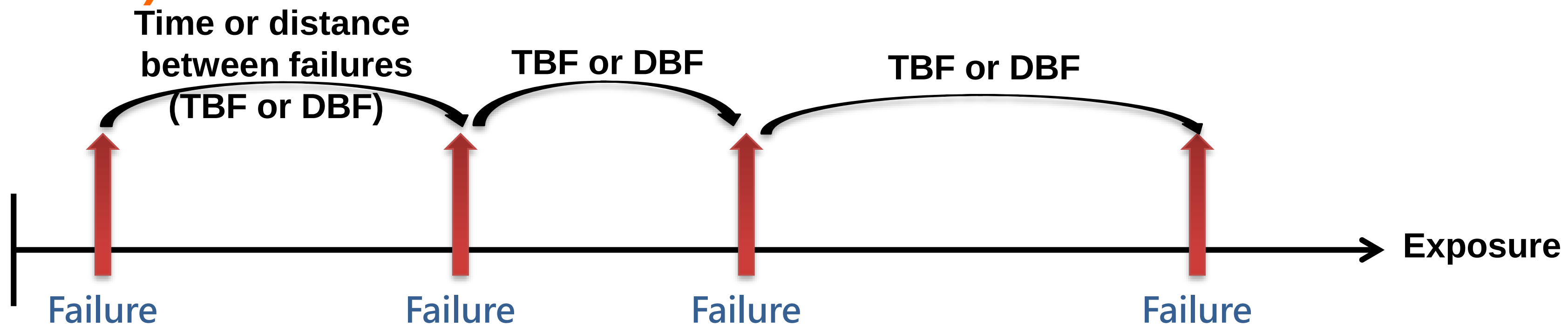
Life Cycle of Railway System (V representation in EN50126)



Picture from: Andrea vn Linn
https://www.linkedin.com/pulse/r-m-roads-railways-data-centers-technical-systems-andreas-van-linn

System reliability cannot reflect its impact to passengers

- Various reliability allocation methods have been developed in the past.
  - **Weighting Method**
  - **Optimization Method**
- For rail system, **reliability**, so called **system reliability**, is defined as the **mean time between failures (MTBF)** or **mean distance between failures (MDBF)**



However, this attribute does not consider its effect on passengers (the consequence of failure)

System Reliability vs. Service Reliability

Minor Train failure	*Communication failure*
MTBF: 100,000 train-hour	**MTBF: 100,000 train-hour**

- ❖ Failure frequency → MTBF → System Reliability
- ❖ Consequence of failure → Delay → Service Reliability

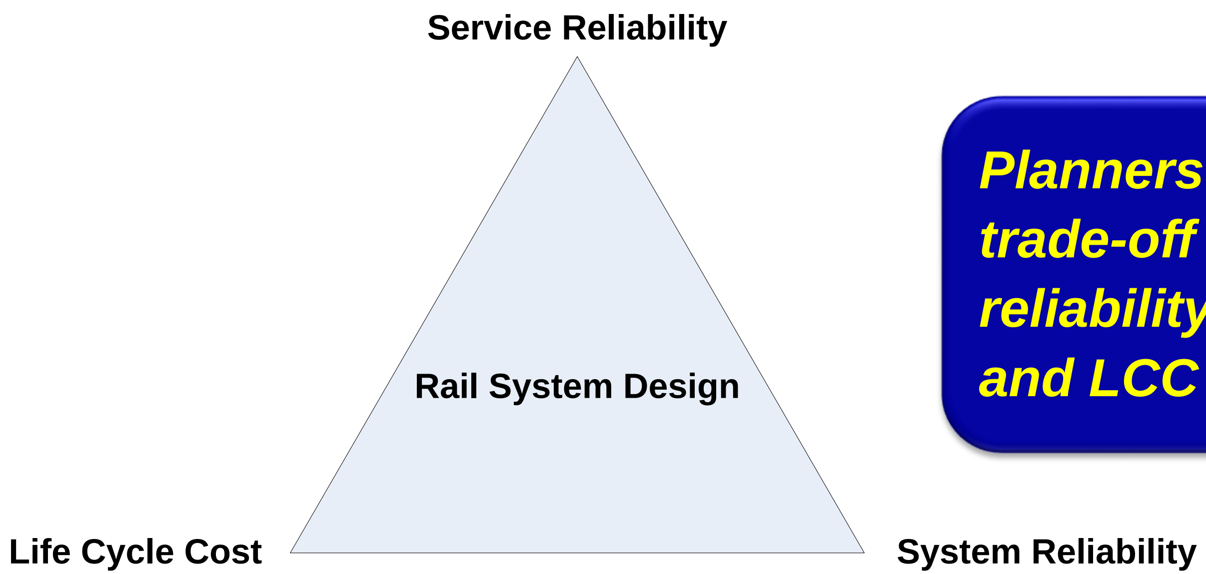
Both of them have same system reliability, but their effects on passengers are quite different.

From System Reliability to Service Reliability

- **Service reliability** (e.g. delay or on-time percentage) is more favorable than system reliability because it considers **customers' satisfaction**
- Service reliability can be obtained by the relationship between service reliability and system reliability

Service Reliability **No. of Failures** **Impact to the customers**

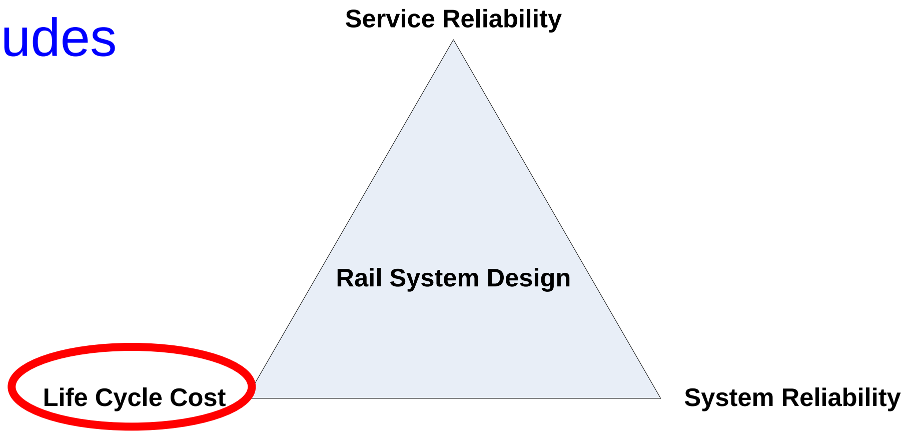
$$Estimated\ Delay_{ik} = \left(\frac{Total\ Operational\ Time}{MTBF_{ik}} \right) \times (Average\ Delay)_{ik}$$



Planners should balance the trade-off among service reliability, system reliability, and LCC in Rail System Design

Key Elements in Rail System Design – LCC

- LCC for railway systems typically includes **capital investment**, **operating cost**, and **maintenance cost** within the planning period

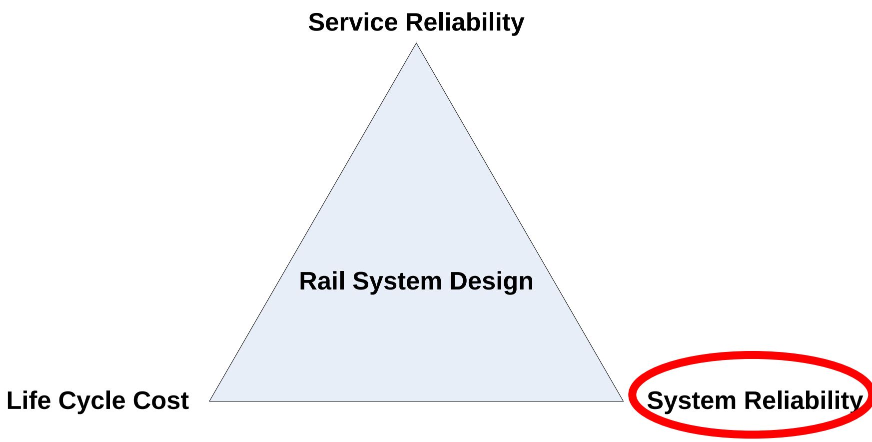


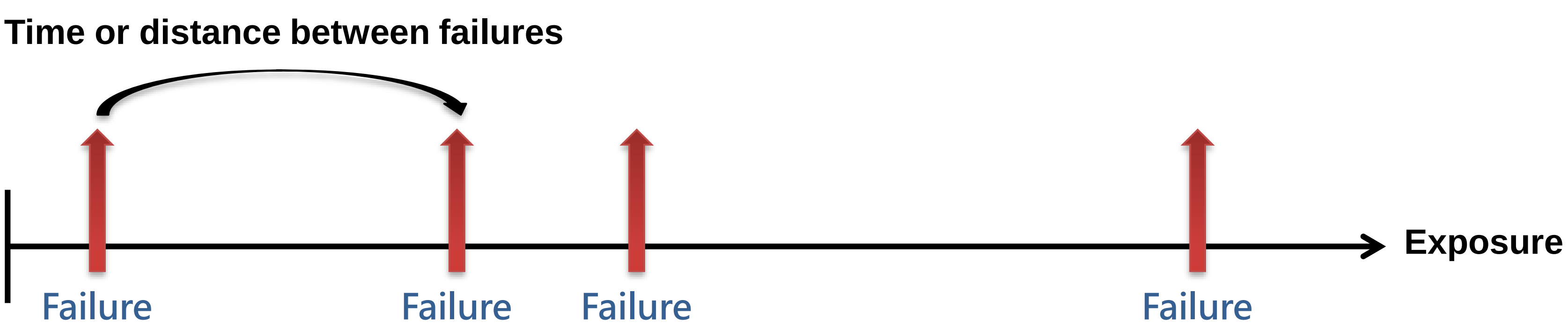
LCC = Capital Cost + Operating Cost + Maintenance Cost

- Employing **LCC** is more appropriate in decision making than solely employing **capital investment**
  - Some products have **low** capital investment but **high** operating and maintenance costs (e.g. ballast track)
  - Others have **high** capital investment but **low** operating and maintenance costs (e.g. slab track)

Key Elements in Rail System Design – System Reliability

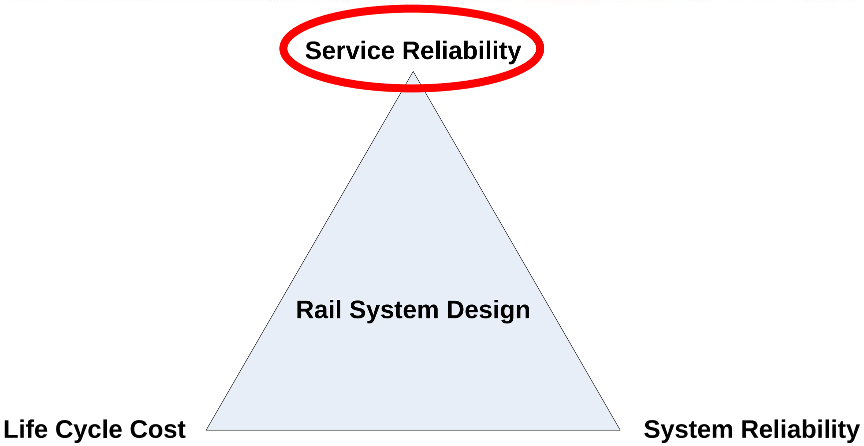
- System reliability is defined as **MTBF** or **MDBF**, and **Failure Rate (= 1/MTBF)**
- The **higher** MTBF or MDBF results in **higher** system reliability
- Information about **system reliability** and **LCC** can be obtained from suppliers





Key Elements in Rail System Design – Service Reliability

- Service reliability identifies **the effect on passengers**
- Target Service Reliability – **On-time arrival percentage** (with no buffer): proportion of on-time operations in terms of total system operating time (in train-hour)



$$\boxed{r_{ser}} = \left(\frac{\boxed{P - \sum_{j \in J} D_j f_j^B}}{\boxed{P}} \right) \times 100\%$$

Total Delay time (train-hour)

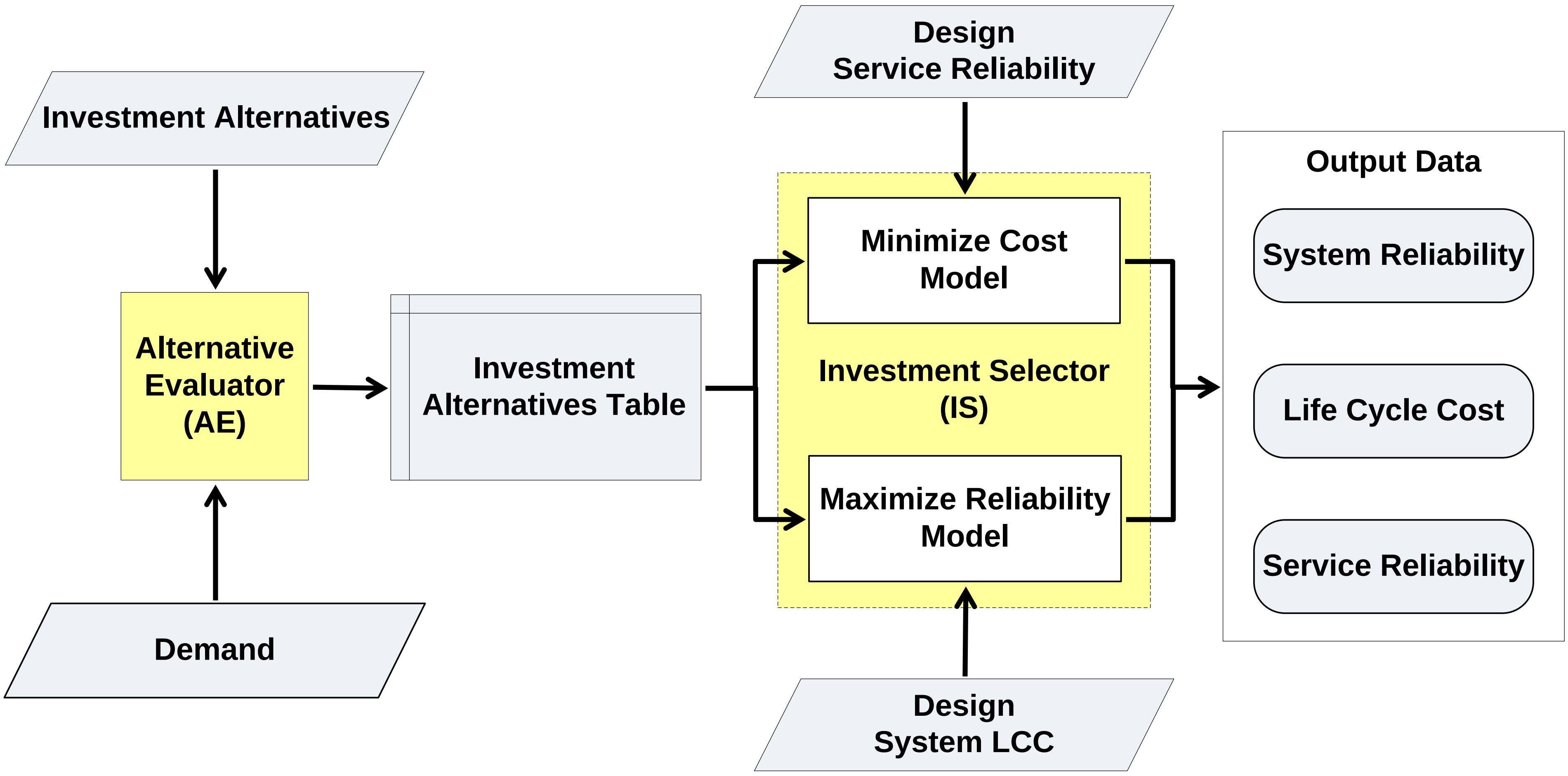
On-time arrival time (train-hour)

Total system operational time (train-hour)

On-time arrival percentage (service reliability)

r_{ser}	=	On-time arrival percentage
P	=	Total system operational time (in train-hour) in a defined period
D_j	=	Delay (in train-hour) of subsystem j within operational time or distance
f_j^B	=	Failure rate

Optimization Framework with MCM and MRM (1st Development)



Alternative Evaluator (AE)

- AE evaluates all possible alternatives and generates an **investment alternative table** with their LCC, system reliability, and service reliability
- Service reliability needs to be computed based on system reliability

Number of failures

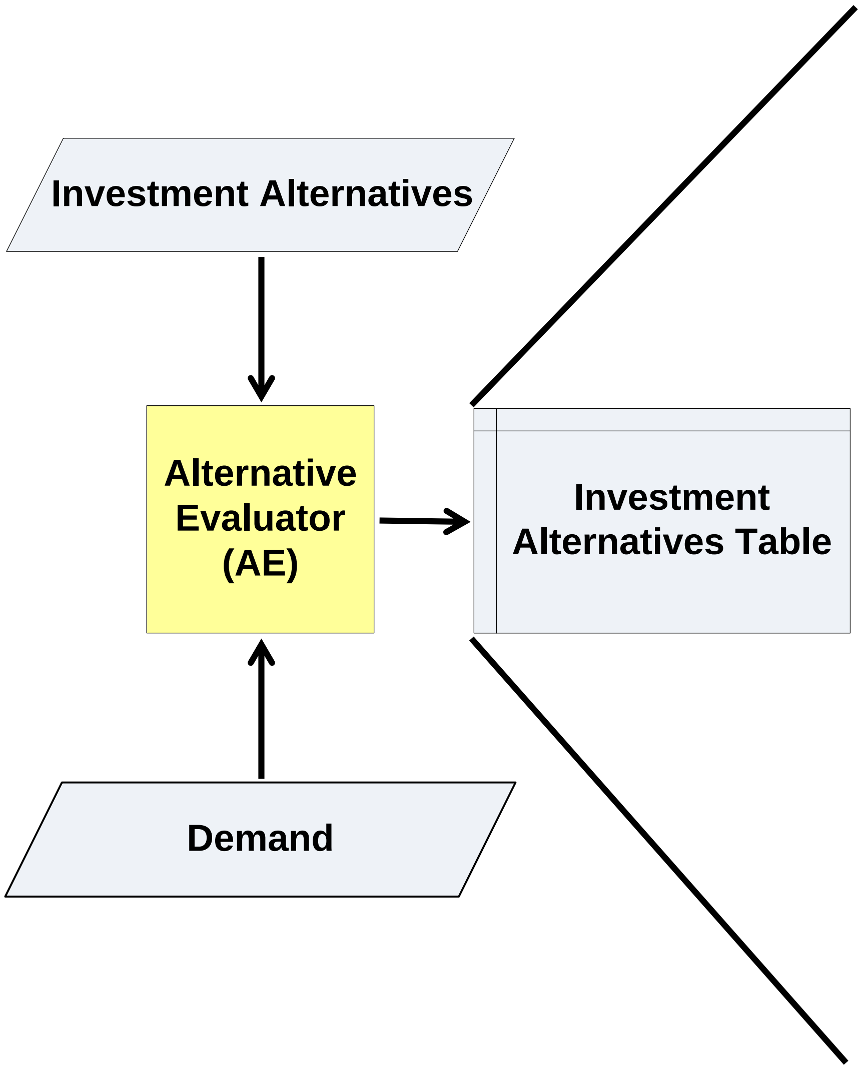
Average train delay

$$D_{ik} = \boxed{\left(\frac{T_i}{M_{ik}}\right)} \boxed{NQ_{ik}} \qquad \forall i \in I, k \in K$$

D_{ik}	=	Delay (in train-hour) of subsystem *i* with alternative *k*
M_{ik}	=	MTBF or MDBF of subsystem *i* with alternative *k*
T_i	=	Operational time or distance of subsystem *i* in a defined period
N	=	Average number of online trains
Q_{ik}	=	Average delay (in hours) from a failure of subsystem *i* with alternative *k*

– Q_{ik} is estimated using historical data from similar systems or determined by simulations based on the service effect from possible types of failures

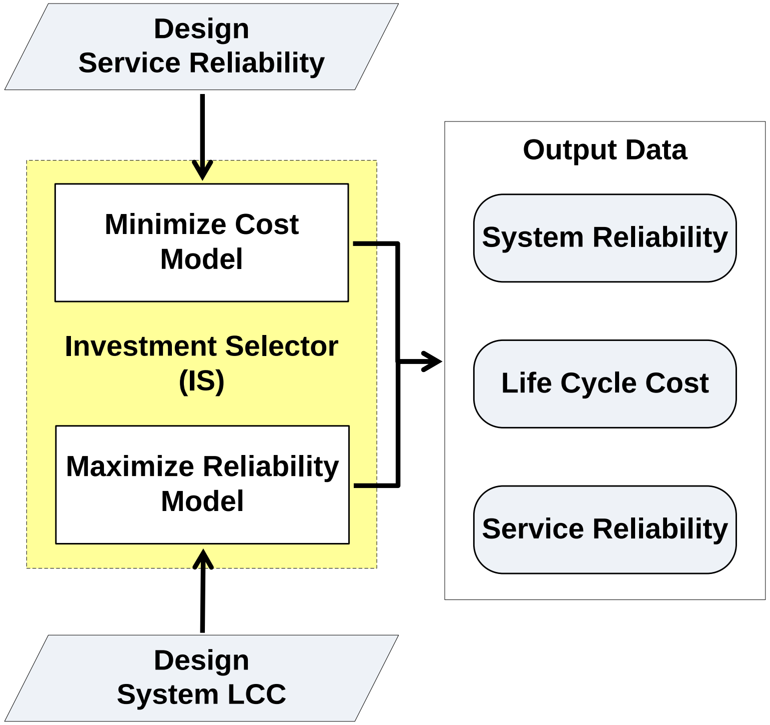
Alternative Evaluator (AE)

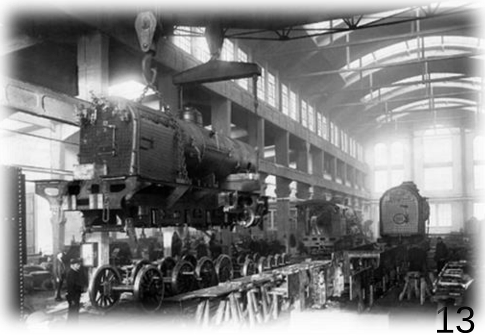


Subsystem (i)	Alternatives (k)	MDBF (M_{ik}) (train-km)	LCC (C_{ik}) billion dollars	Delay (D_{ik}) (train-hours)
Train	1	29,274	16.63	393
	2	39,799	16.76	289
	3	62,192	17.05	185
	⋮	⋮	⋮	⋮
Electricity	1	29,274	20.86	2,144
	2	38,521	22.48	1,629
	3	48,133	24.32	1,304
	⋮	⋮	⋮	⋮
Track	1	36,716	14.81	1,363
	2	56,073	15.31	892
	3	64,959	17.05	770
	⋮	⋮	⋮	⋮
Signal	1	29,274	30.03	1,110
	2	31,039	31.01	1,047
	3	39,799	36.45	817
	⋮	⋮	⋮	⋮
Communication	1	36,716	8.56	3,395
	2	53,899	9.75	2,313
	3	78,465	11.77	1,589
	⋮	⋮	⋮	⋮
Station	1	67,941	3.91	277
	2	84,846	1.95	222
	3	123,063	2.03	153
	⋮	⋮	⋮	⋮

Investment Selector (IS)

- IS identifies the best alternative for every subsystem according to acceptable LCC or service reliability
  - **Minimize Cost Model (MCM):** Minimizing total LCC according to acceptable service reliability
  - **Maximize Reliability Model (MRM):** Maximizing service reliability according to available LCC





Minimize Cost Model

Min $\sum_{i\in I}\sum_{k\in K} C_{ik}\delta_{ik}$ → **Minimize total LCC**

S.t. $\sum_{k\in K}\delta_{ik} = 1 \quad \forall i\in I$ → **Only one alternative can be chosen for a subsystem**

$d_i = \sum_{k\in K} D_{ik}\delta_{ik} \quad \forall i\in I$ → **Compute delay for each subsystem**

$\left(\frac{P-\sum_{i\in I} d_i}{P}\right)\times 100\% \geq R$ → **Fulfill the service reliability requirement**

and

$\delta_{ik}\in\{0,1\} \quad \forall i\in I, k\in K$

$d_i \geq 0 \quad \forall i\in I$

Decision Variables	
δ_{ik}	whether alternative is selected
d_i	the delay of subsystem *i*
Parameters	
C_{ik}	LCC of alternative
D_{ik}	delay of alternative
P	total operational time per year
R	design service reliability

Maximize Reliability Model

Max $\left(\dfrac{P - \sum_{i \in I} d_i}{P} \right) \times 100\%$ → **Maximize service reliability**

S.t.

$$\sum_{k \in K} \delta_{ik} = 1 \qquad \forall i \in I$$

→ **Only one alternative can be chosen for a subsystem**

$$d_i = \sum_{k \in K} D_{ik} \delta_{ik} \qquad \forall i \in I$$

→ **Compute delay for each subsystem**

$$\sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \le B$$

→ **Constraint on LCC**

and

$$\delta_{ik} \in \{0,1\} \qquad \forall i \in I, k \in K$$

$$d_i \ge 0 \qquad \forall i \in I$$

Decision Variables	
δ_{ik}	whether alternative is selected
d_i	the delay of subsystem *i*
Parameters	
C_{ik}	LCC of alternative
D_{ik}	delay of alternative
P	total operational time per year
B	design LCC

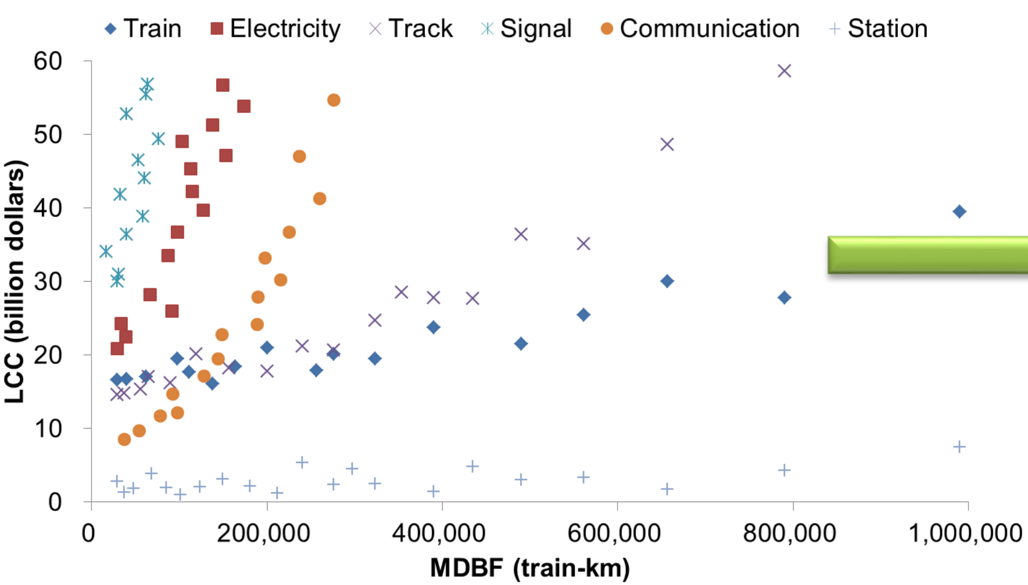
Two case studies to demonstrate the potential use

- Two case studies with empirical data obtained from a rail system in Taiwan were performed to show the potential use of the proposed method
  - **Case I : New System Design**
    - Designing a new passenger rail system
    - Selecting appropriate alternatives for subsystems according to **design service reliability → MCM**
  - **Case II : Existing System Improvements**
    - Improving the reliability of an existing rail system
    - Subject to constraint on **available increment in LCC → MRM**

Case I : New System Design

- 25-km passenger rail system
- Estimated demand is 140,000 passengers per day
- Six subsystems : train, signal, communication, electricity, station, and track
- Design service reliability (on-time arrival percentage) is from **95% to 99%,** with 1% increments

Investment Alternatives



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	⋮	⋮	⋮	⋮

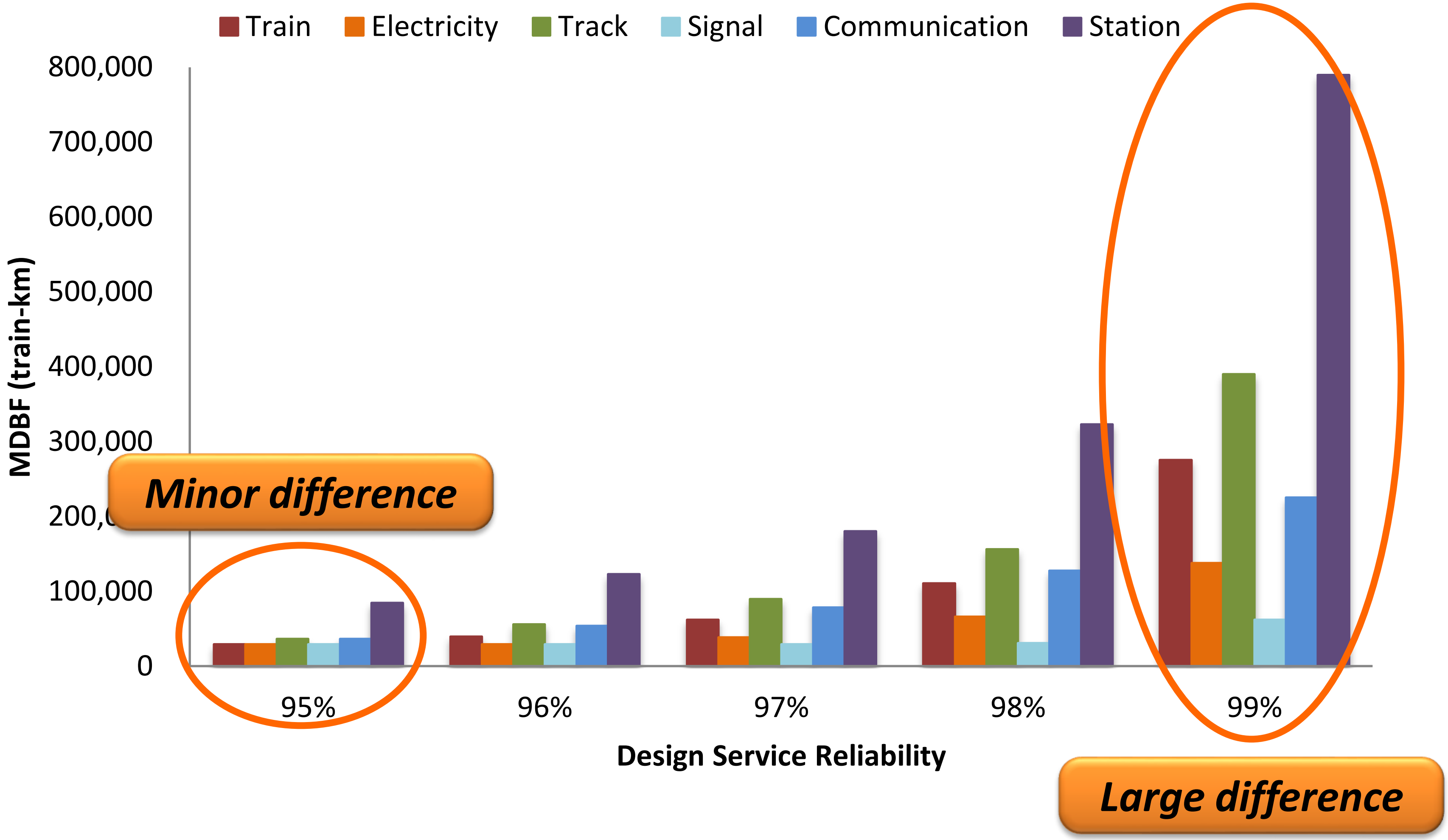
High design service reliability results in high MDBF and LCC

❖ MCM efficiently solved this problem by using CPLEX within seconds

	Subsystem	Design Service Reliability				
		95%	96%	97%	98%	99%
MDBF (train-km)	Train	29,274	39,799	62,192	110,912	275,593
	Electricity	29,274	29,274	38,521	66,422	137,907
	Track	36,716	56,073	89,628	156,454	389,915
	Signal	29,274	29,274	29,274	31,039	62,192
	Communication	36,716	53,899	78,465	127,670	225,146
	Station	84,846	123,063	180,291	323,230	789,958
LCC (billion dollars)	Train	16.63	16.76	17.05	17.71	20.18
	Electricity	20.86	20.86	22.48	28.28	51.37
	Track	14.81	15.31	16.24	18.28	27.85
	Signal	30.03	30.03	30.03	31.01	55.42
	Communication	8.56	9.75	11.77	17.21	36.79
	Station	1.95	2.03	2.17	2.54	4.34
	Total	92.83	94.75	99.74	115.04	195.94

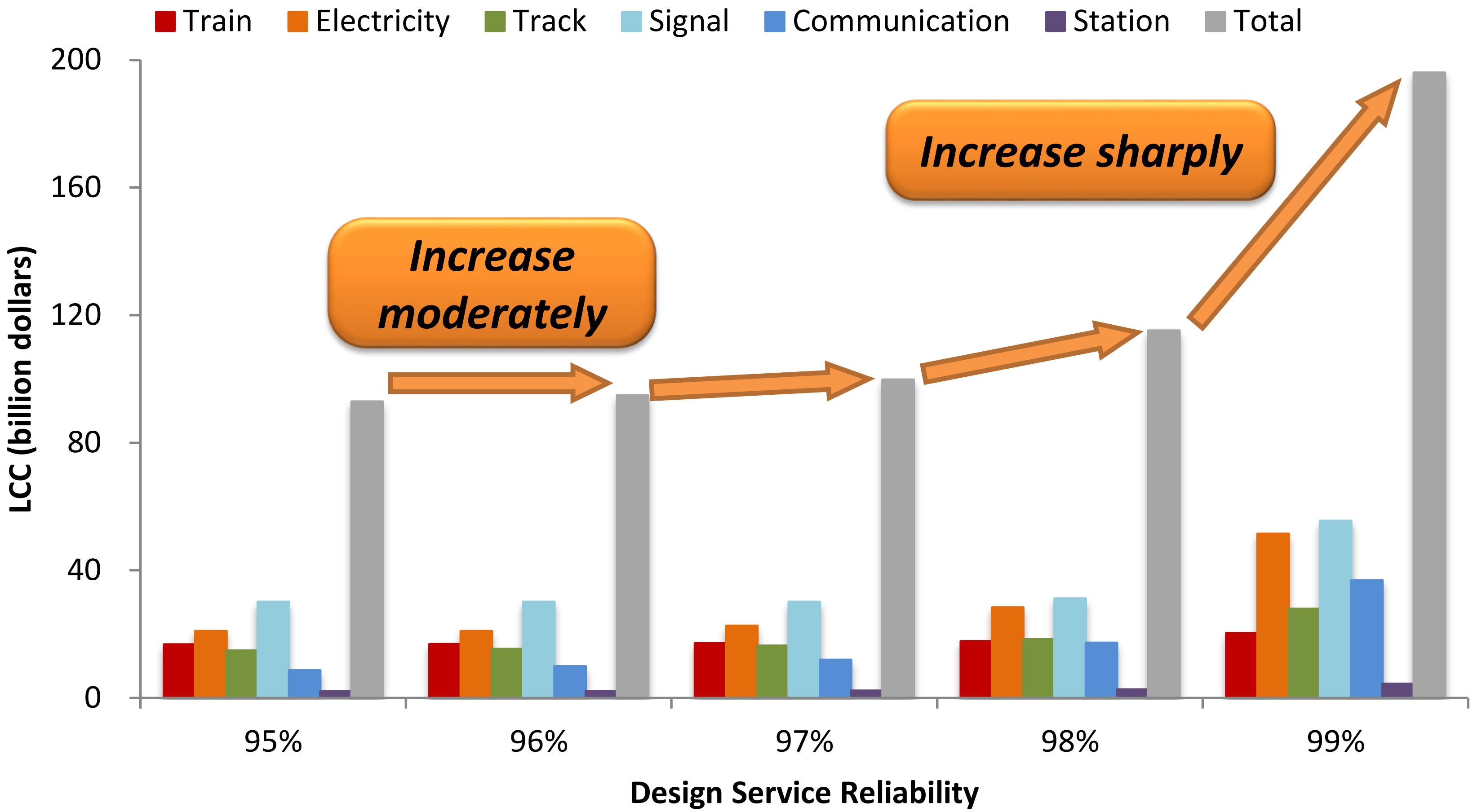
Resulting MDBF from MCM

❖ The difference in MDBF among subsystems becomes obvious as service reliability level increase



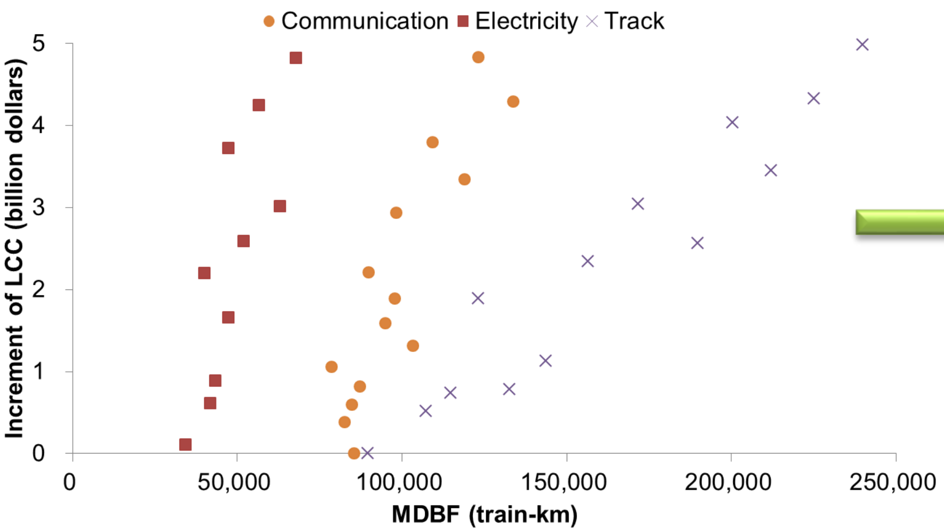
Resulting LCC from MCM

- Increase in total LCC from 95% to 97% is modest but very sharp from 97% to 99% because of the nonlinear relationship between cost and reliability



Case II :
Existing System Improvement

- Demand is the same as Case I and service reliability of the existing system is **97%** with improvement **LCC from 1 ~ 5 billions**
- Not all of the subsystems can be easily changed so we consider alternatives for **communication**, **electricity**, and **track** in this case
- MRM efficiently solved this problem by using CPLEX within seconds



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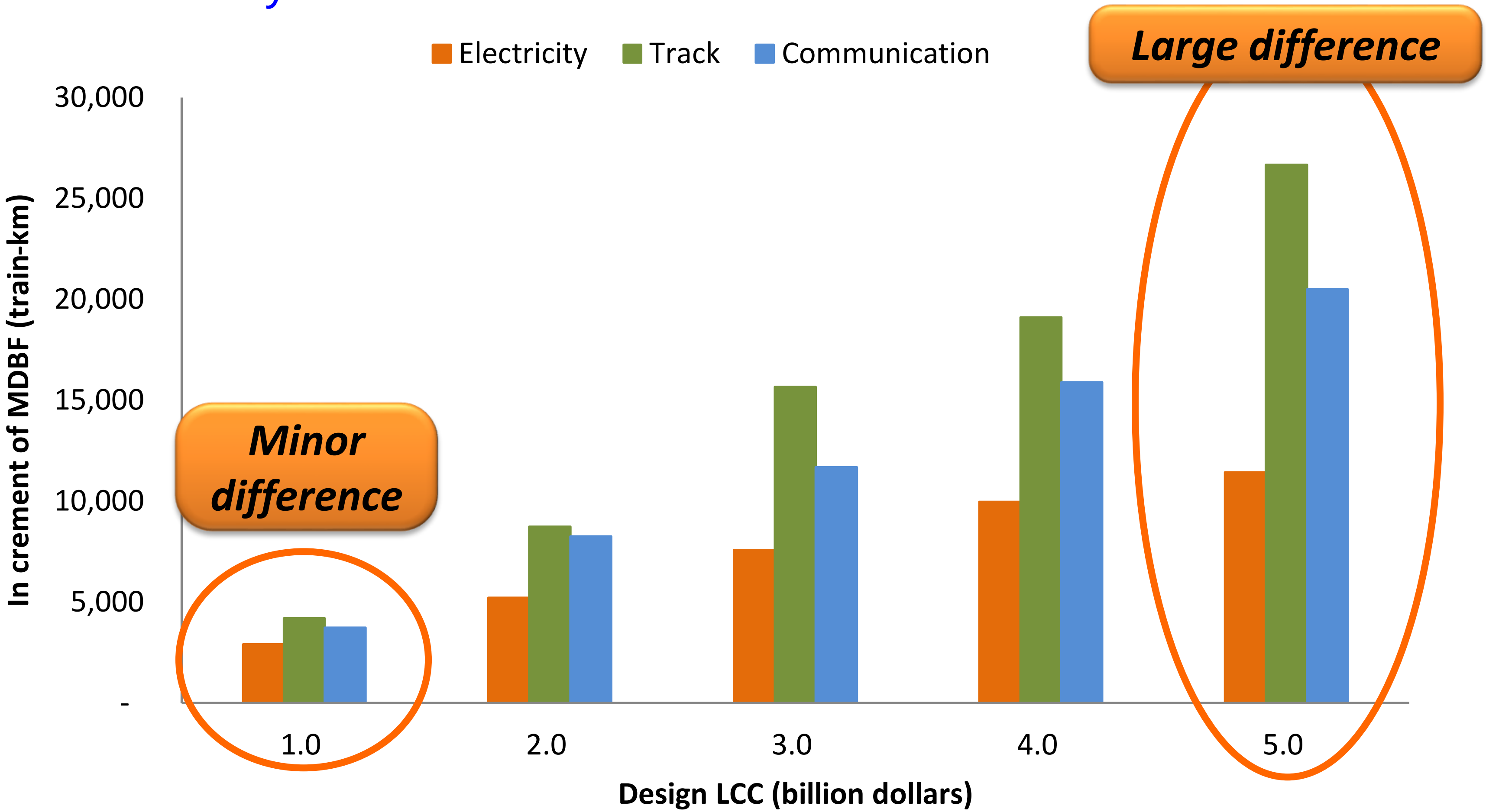
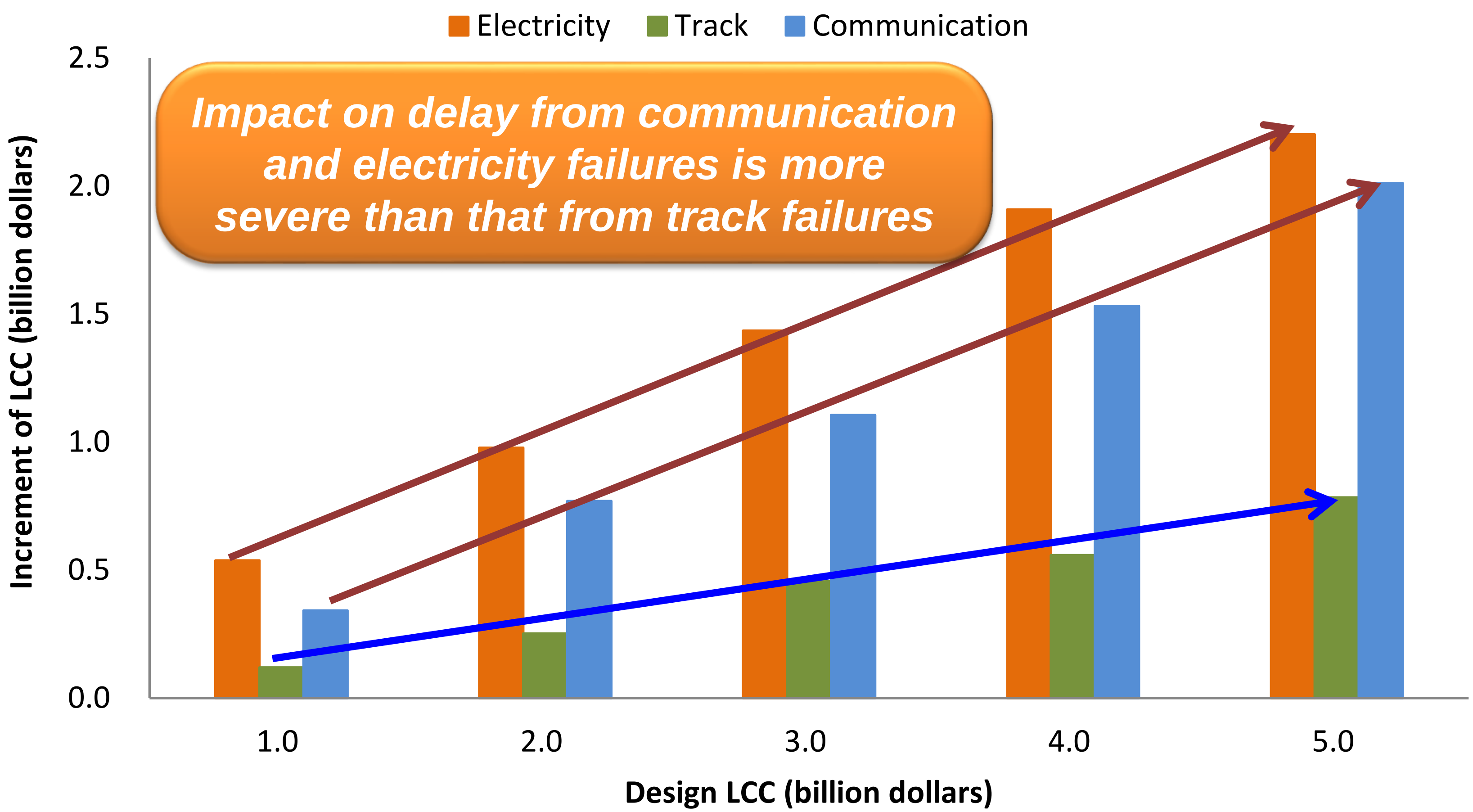
Resulting MDBF from MRM

❖ Track has the most significant increase in MDBF, followed by communication and electricity

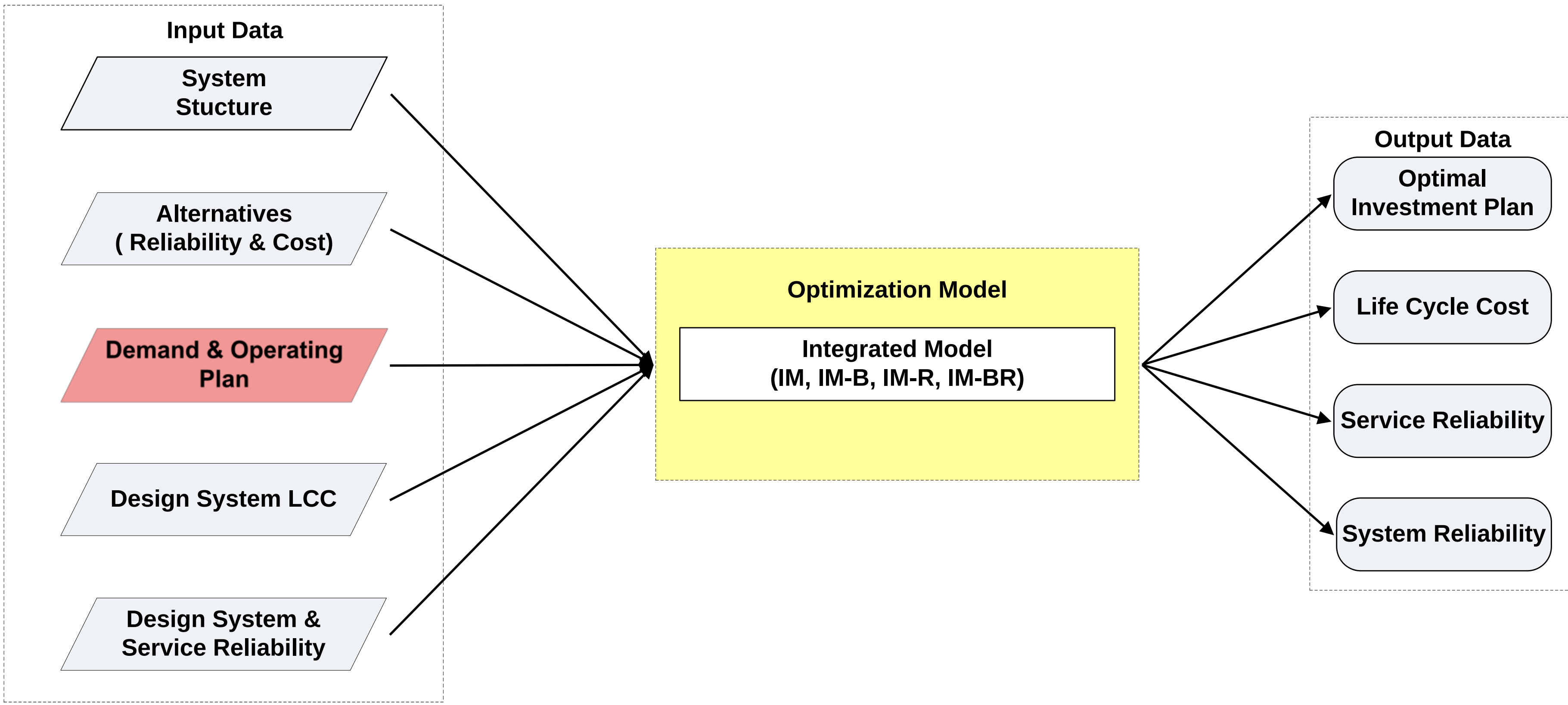


Resulting LCC from MRM

- More LCC have been allocated to electricity, and communication for all scenarios



Optimization Framework with Integrated Model (2nd Development)



An integrated optimization framework for rail system design is developed by minimizing LCC and service unreliability (delay cost)

Integrated Model – Objective Function

❖ The objective function considers the trade-off between cost and reliability, by minimizing total LCC and delay cost

$$\textit{Min} \quad \underbrace{\boxed{\sum_{i\in I}\sum_{n\in N} C_{in}\delta_{in}}}_{\text{LCC}} + \underbrace{\boxed{HT\sum_{j\in J_r} D_j f_j^B}}_{\text{Delay cost}}$$

Integrated Model

- ***Original Model (IM)***
- ***With Budget (IM-B)***
- ***With Reliability (IM-R)***
- ***With Both (IM-BR)***

$\boldsymbol{\delta_{in}}$ **a binary variable denoting the selection of the alternative**

$\boldsymbol{f^B{}_j}$ **a continuous variable denoting the failure rate of subsystem *j***

Types of Constraints

Group 1 – Alternatives Selection

$$\sum_{n \in N_i} \delta_{in} = 1 \qquad \forall i \in I, N_i \in N$$

$$\sum_{k \in K} y_{jk} = 1 \qquad \forall j \in J_r$$

$$\sum_{(i,n) \in V_k} \delta_{in} \le y_{jk} + (U_j - 1) \qquad \forall k \in K_j, j \in J_r$$

Group 2 – Failure Rate Computation

$$f_{jk}^{C} \ge F_{in}^{C} + m(1 - y_{jk}) \qquad \forall (i,n) \in V_k, k \in K_j, j \in J_s$$

$$f_{jk}^{C} \ge \prod_{(i,n) \in V_k} F_{in}^{C} + m(1 - y_{jk}) \quad \forall k \in K_j, j \in J_p$$

Group 3 – System Failure Rate

$$f_j^B \ge f_{jk}^C \qquad \forall k \in K_j, j \in J_r$$

$$f \ge f_j^B \qquad \forall j \in J_r$$

Group 4 – System Requirements

$$\sum_{k \in K} f_{jk}^{m} y_{jk} \le G_j \quad \forall j \in J_r$$

$$f \le E$$

$$\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \le B$$

$$\left(\frac{P - \sum_{j \in J_r} D_j f_j^B}{P} \right) \times 100\% \ge R$$

Group 1 – Alternatives Selection

$$\sum_{n \in N_i} \delta_{in} = 1 \qquad \forall i \in I, N_i \in N$$

→ **Component Selection**

$$\sum_{k \in K} y_{jk} = 1 \qquad \forall j \in J_r$$

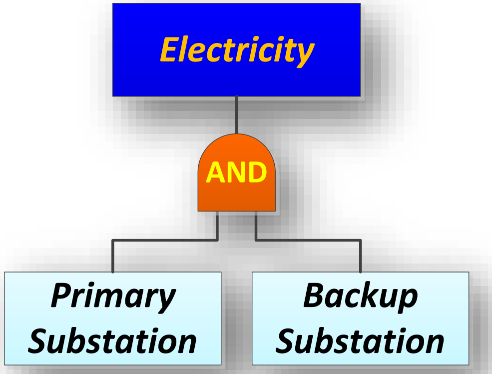
→ **Combination Selection**

$$\sum_{(i,n) \in V_k} \delta_{in} \le y_{jk} + (U_j - 1) \qquad \forall k \in K_j, j \in J_r$$

→ **Relation between Combination and Component Selection**

Subsystem *(j)*	Component Alternatives *(i)*	*(n)*	Failure Rate *(F^C_{in})*	LCC(C_{in}) *(billion dollars)*	Delay (D_j) *(train-hours)*
Electricity	Primary	1	0.16	13.29	3137.82
		2	0.11	19.35	
		⋮	⋮	⋮	
	Backup	1	0.25	4.24	
		2	0.16	5.69	
		⋮	⋮	⋮	

Combination I



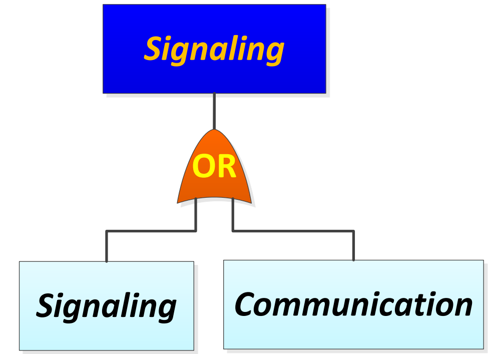
δ_{in} ***a binary variable denoting the selection of the alternative***

y_{jk} ***a binary variable denoting the selection of the combination***

Group 2 –
Failure Rate Computation

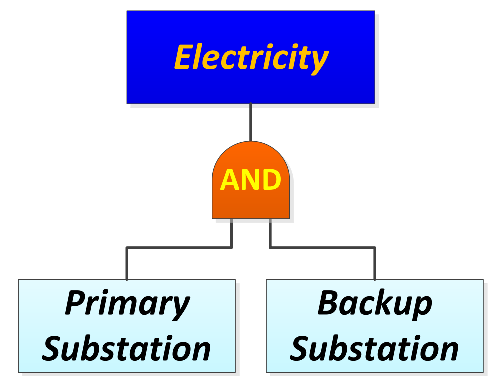
$$f_{jk}^{C} \geq F_{in}^{C} + m(1 - y_{jk}) \qquad \forall (i,n) \in V_k, k \in K_j, j \in J_s$$

- **Failure Computation for Components with "OR" Relationship**



$$f_{jk}^{C} \geq \prod_{(i,n)\in V_k} F_{in}^{C} + m(1 - y_{jk}) \quad \forall k \in K_j, j \in J_p$$

- **Failure Computation for Components with "AND" Relationship**



y_{jk} ***a binary variable denoting the selection of the combination***

f^{C}_{jk} ***a continuous variable denoting the failure rate of each combination k in subsystem j***

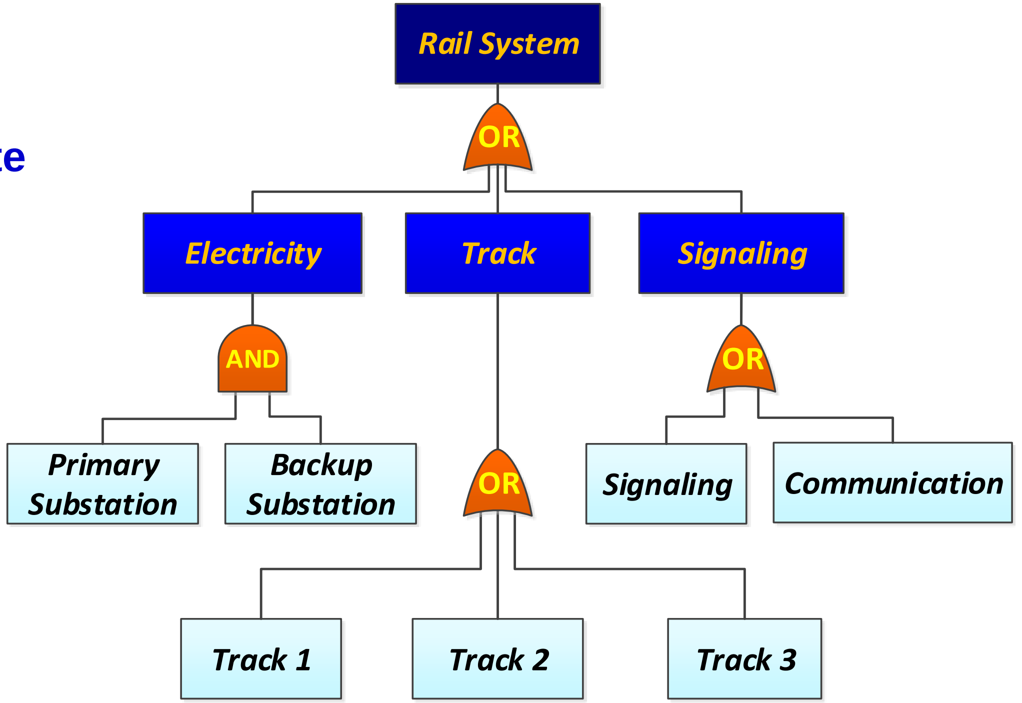
Group 3 –
System Failure Rate

$$f_j^B \geq f_{jk}^C \qquad \forall k \in K_j, j \in J_r$$

→ **Returning the Subsystem Failure Rate**

$$f \geq f_j^B \qquad \forall j \in J_r$$

→ **Returning the System Failure Rate**



$f^C{}_{jk}$ ***a continuous variable denoting the failure rate of each combination k in subsystem j***

$f^B{}_j$ ***a continuous variable denoting the failure rate of subsystem j***

f ***a continuous variable denoting the failure rate of the system***

Group 4 –
System Requirements

$$f \le E$$

→ Constraint on total system reliability

$$\sum_{k \in K} f^{C}_{jk} y_{jk} \le G_j \quad \forall j \in J_r$$

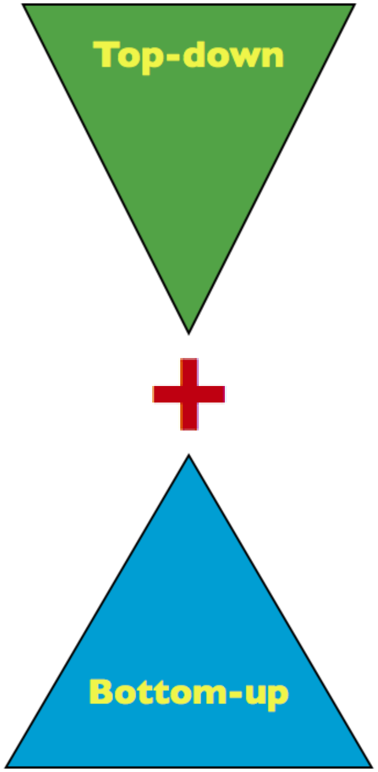
→ Constraint on
system reliability for each subsystem

$$\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \le B$$

→ Constraint on LCC budget

$$\left(\frac{P - \sum_{j \in J_r} D_j f^{B}_{j}}{P} \right) \times 100\% \ge R$$

→ Constraint on service reliability

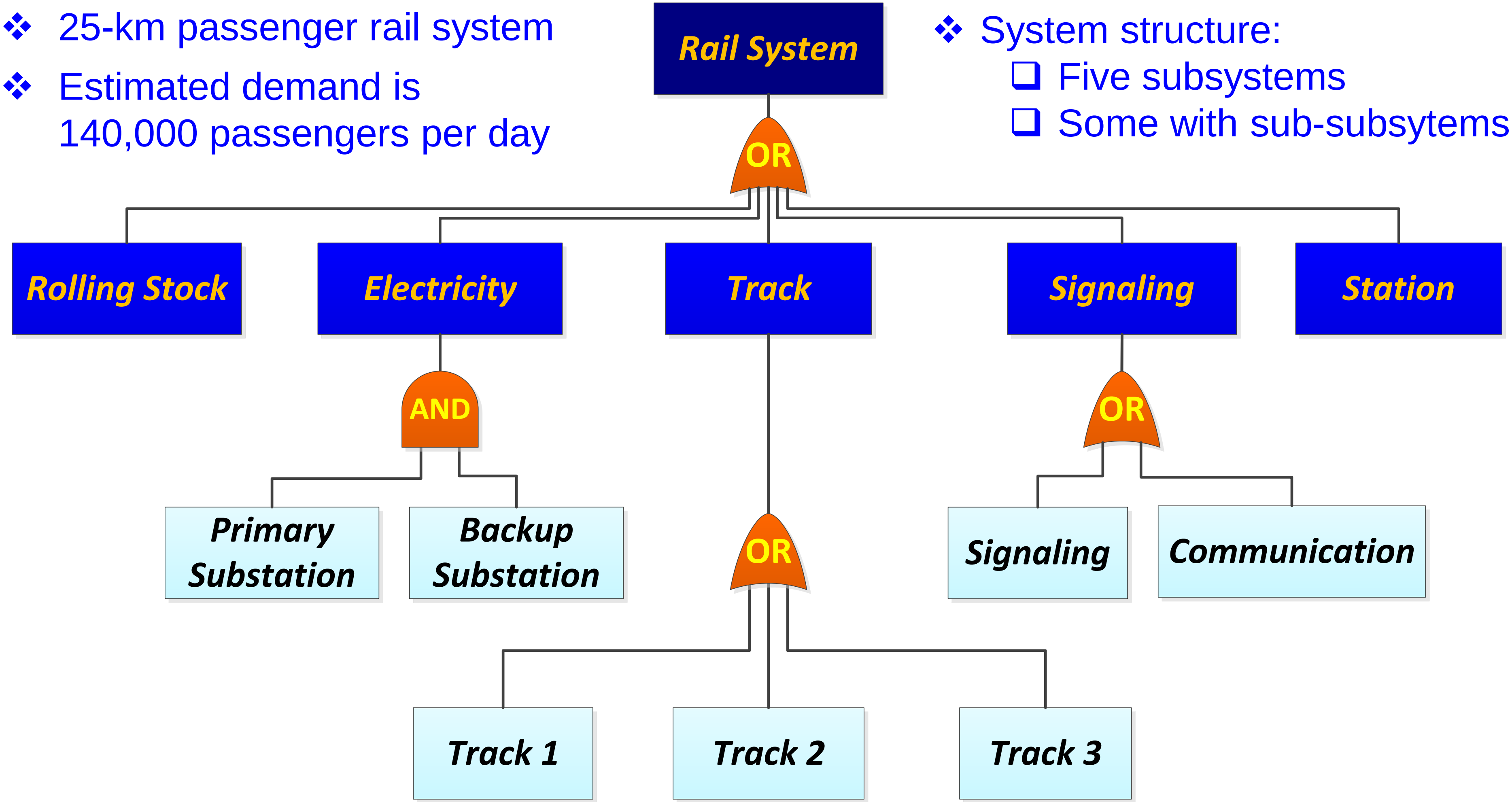


IM-B ***: IM + Constraint of LCC***
IM-R ***: IM + Constraint of service reliability***
IM-BR ***: IM + Constraints of LCC and service reliability***

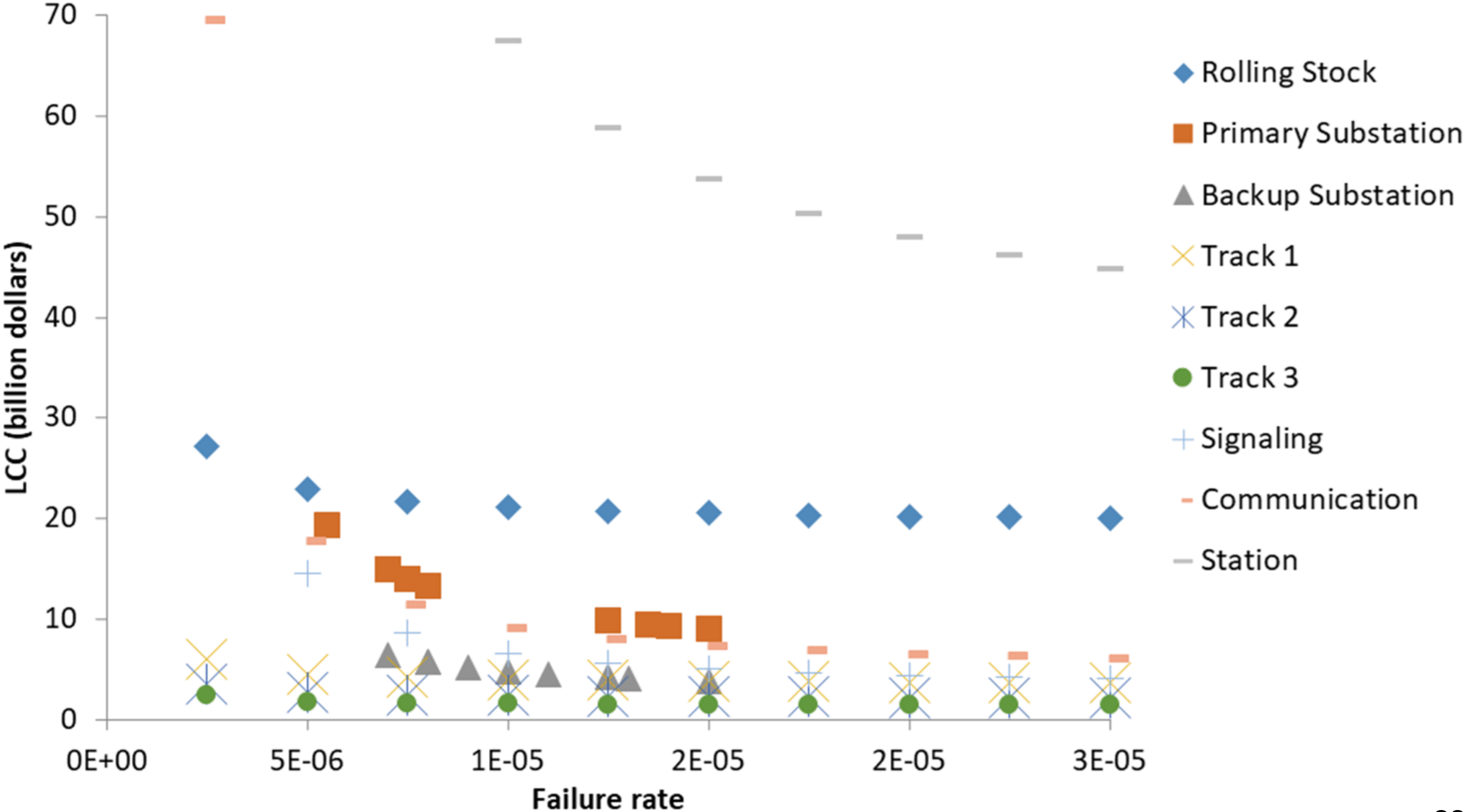
Case Study –
New System Design

- 25-km passenger rail system
- Estimated demand is 140,000 passengers per day

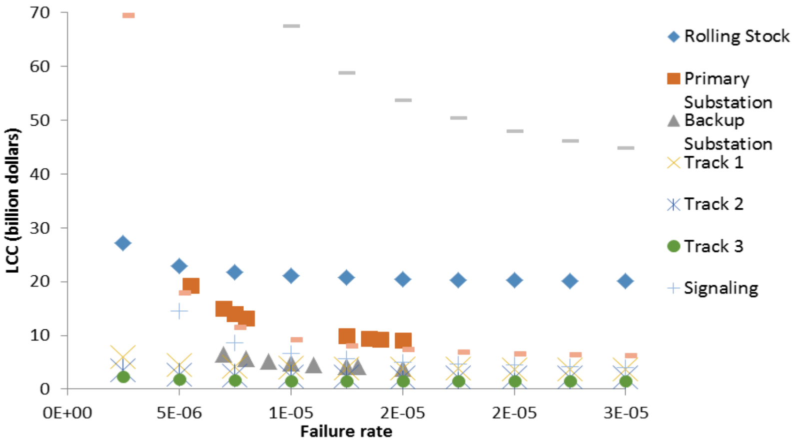
- System structure:
  - Five subsystems
  - Some with sub-subsytems



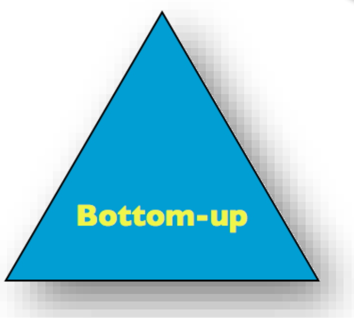
Alternatives with Specific Cost and System Reliability Information



Investment Alternatives Table



Investment Alternatives



Subsystem (j)	Component (i)	Alternatives (n)	Failure Rate (F^{C}_{in})	LCC(C_{in}) (billion dollars)	Delay (D_j) (train-hours)
Rolling Stock	Rolling	1 2 :	0.05 0.1 :	27.21 22.94 :	575.66
Electricity	Primary	1 2 :	0.16 0.11 :	13.29 19.35 :	3137.82
	Backup	1 2 :	0.25 0.16 :	4.24 5.69 :	
Track	Track 1	1 2 :	0.05 0.1 :	5.98 4.57 :	2501.52
	Track 2	1 2 :	0.05 0.1 :	3.59 2.74 :	
	Track 3	1 2 :	0.05 0.1 :	2.39 1.83 :	
Signaling	Signaling	1 2 :	0.05 0.1 :	73.50 14.60 :	6232.20
	Communication	1 2 :	0.05 0.1 :	69.38 17.74 :	
Station	Station	1 2 :	0.05 0.1 :	542.45 134.12 :	942.15

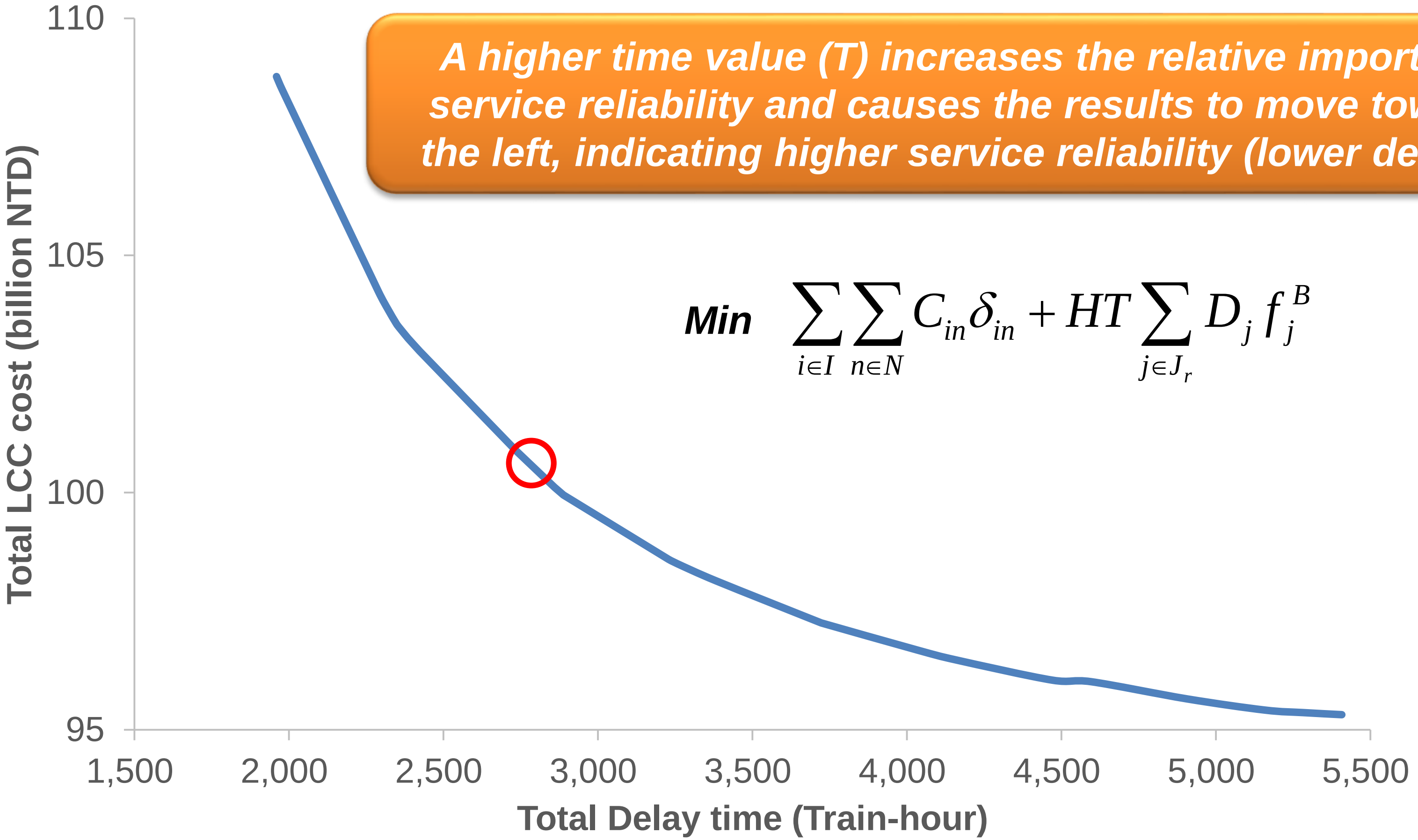
Optimal Investment Plan

- IM efficiently solved this problem by using CPLEX within seconds
- The resulting LCC is $97.25 billion NTD, the total delay cost is $7.23 billion NTD, and service reliability in on-time arrival rate is 97.84%.
- This allocation demonstrates the optimal balance between LCC and service reliability at a given **design time value (*T* = 64,742 NTD/train-hour)**

Subsystem	Failure rate	LCC (billion NTD)	Delay cost (billion NTD)
Rolling stock	2.50E-05	20.06	0.56
Electricity	4.50E-06	12.94	0.55
Track	1.00E-05	8.00	0.97
Signaling	1.75E-05	11.51	4.24
Stations	2.50E-05	44.75	0.92
Total		97.25	7.23

Pareto Front of Optimal Allocation

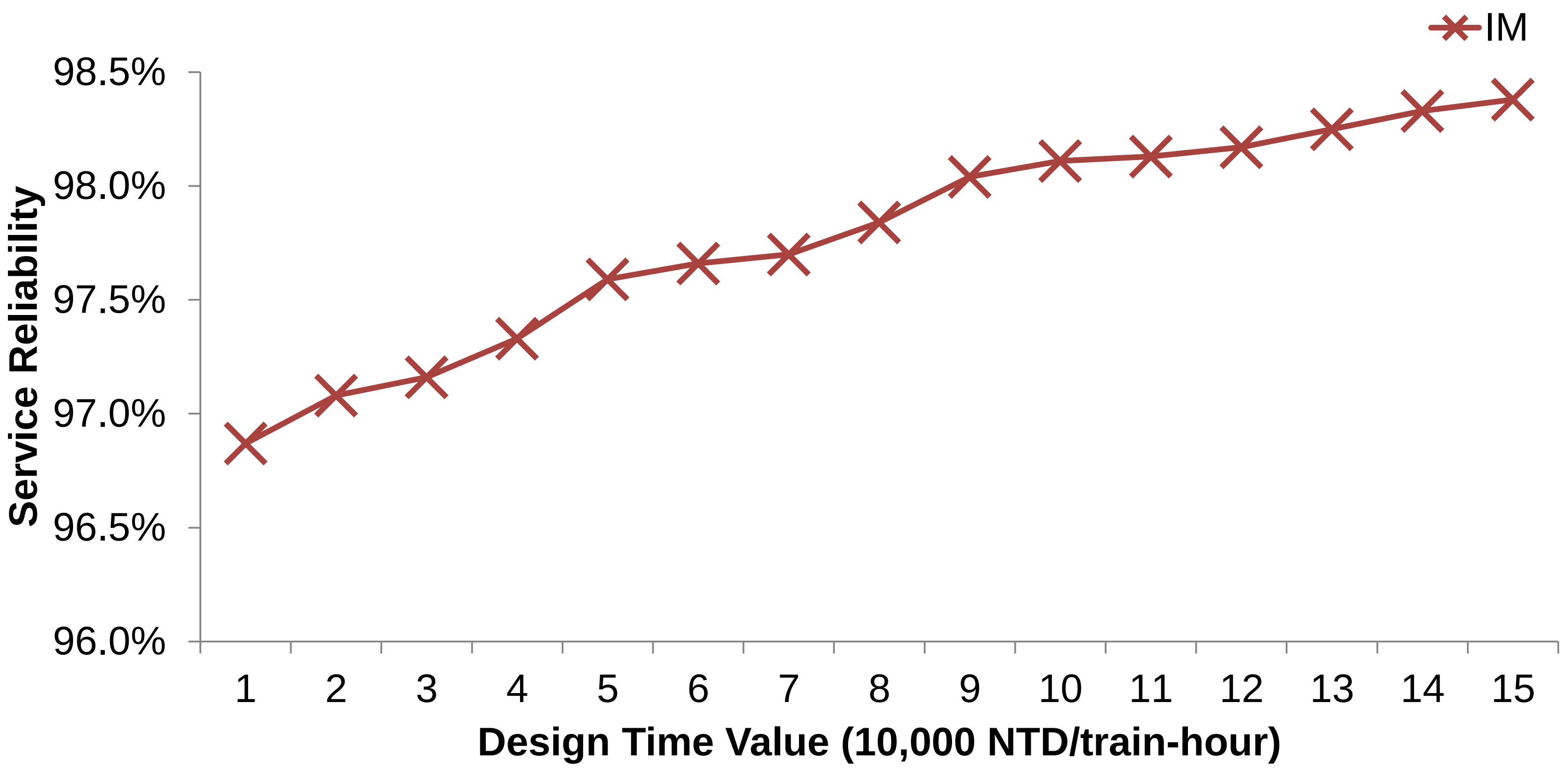
- Each point is the optimal balance between LCC and service reliability at a specific time value.



A higher time value (T) increases the relative importance of service reliability and causes the results to move towards to the left, indicating higher service reliability (lower delay cost)

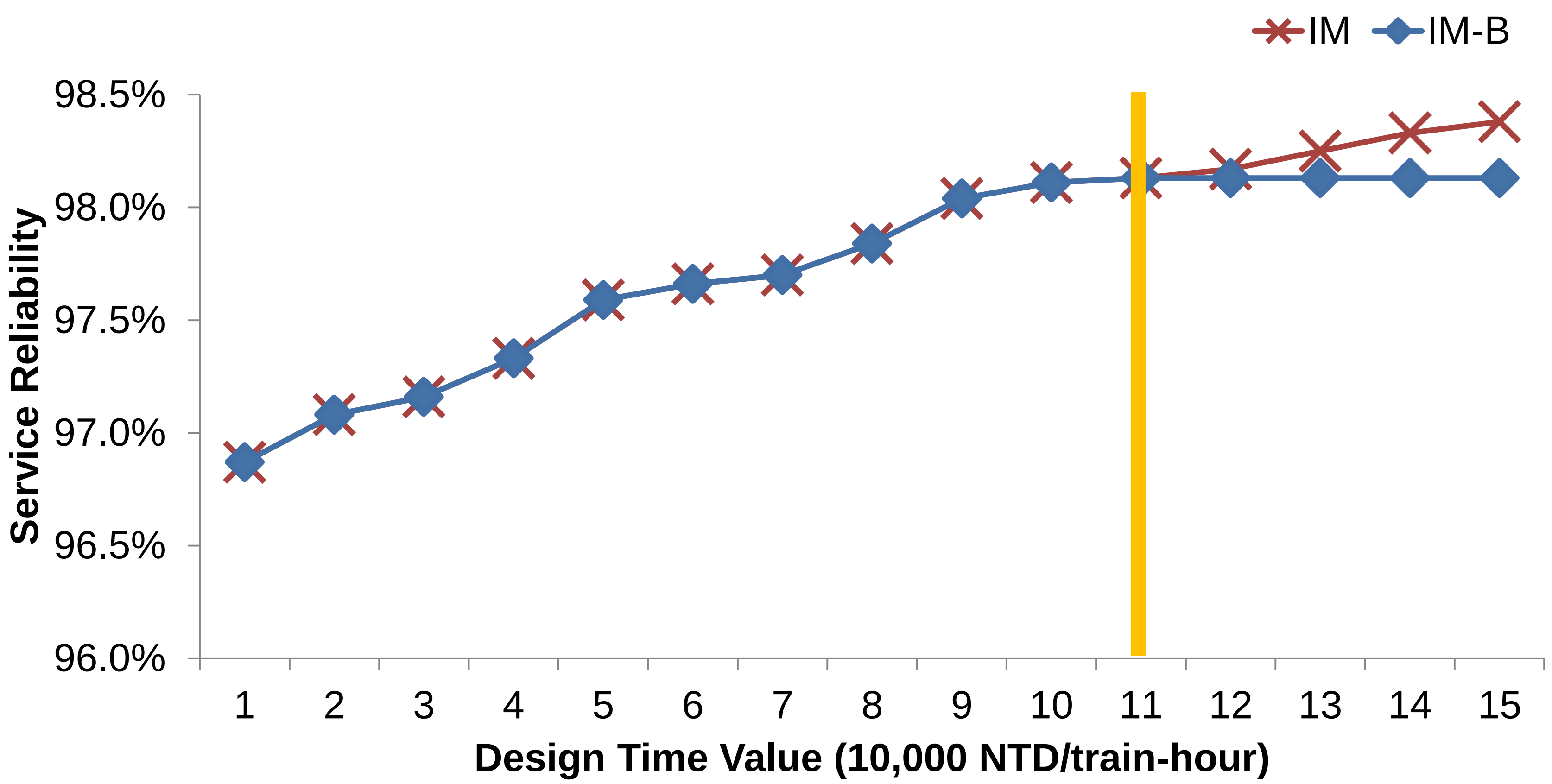
$$\textbf{Min} \quad \sum_{i\in I}\sum_{n\in N} C_{in}\delta_{in} + HT\sum_{j\in J_r} D_j f_j^B$$

The performance of IM with different design time value

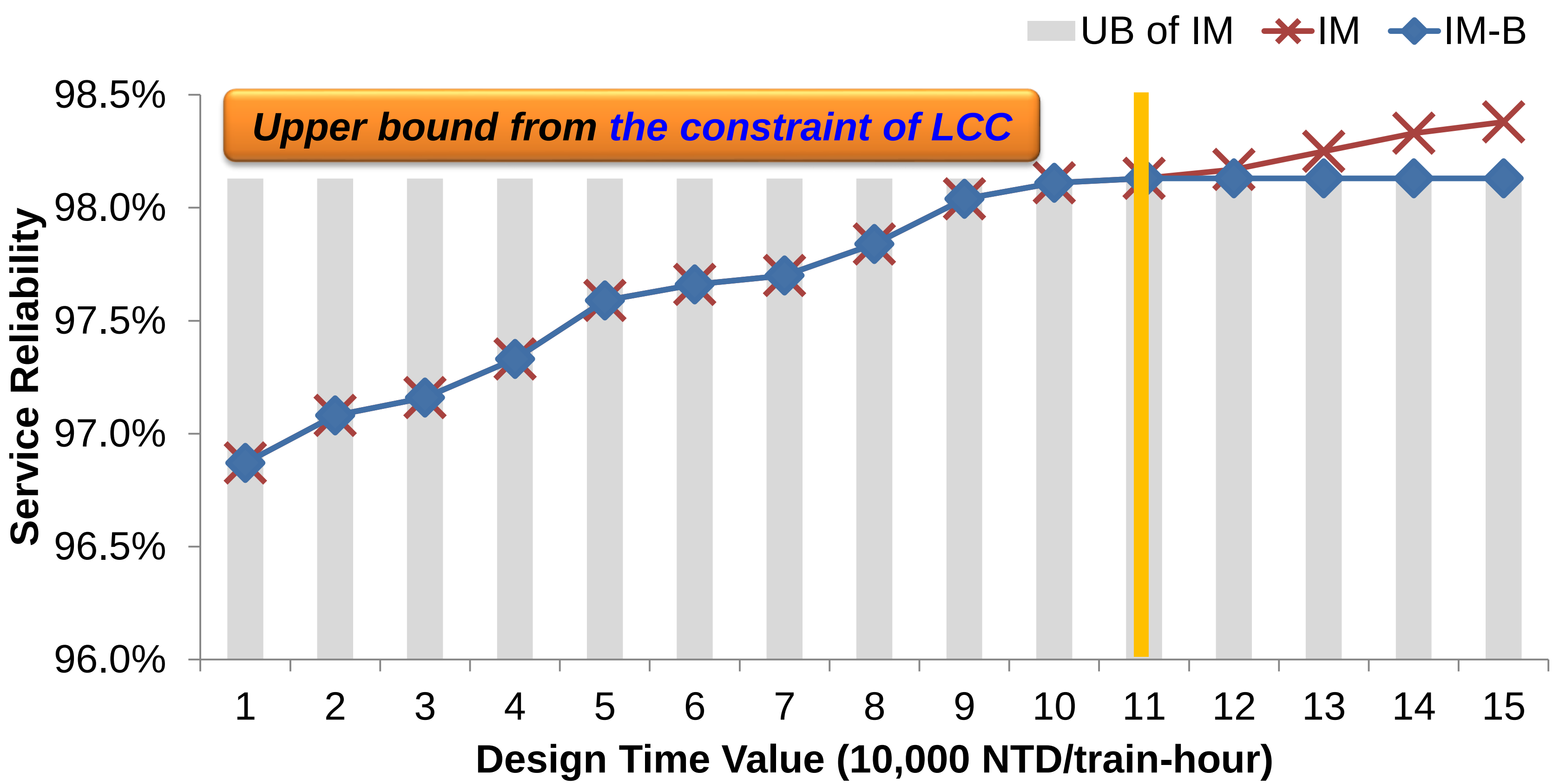


Higher time value results in higher service reliability

IM-B will be constrained by LCC budget

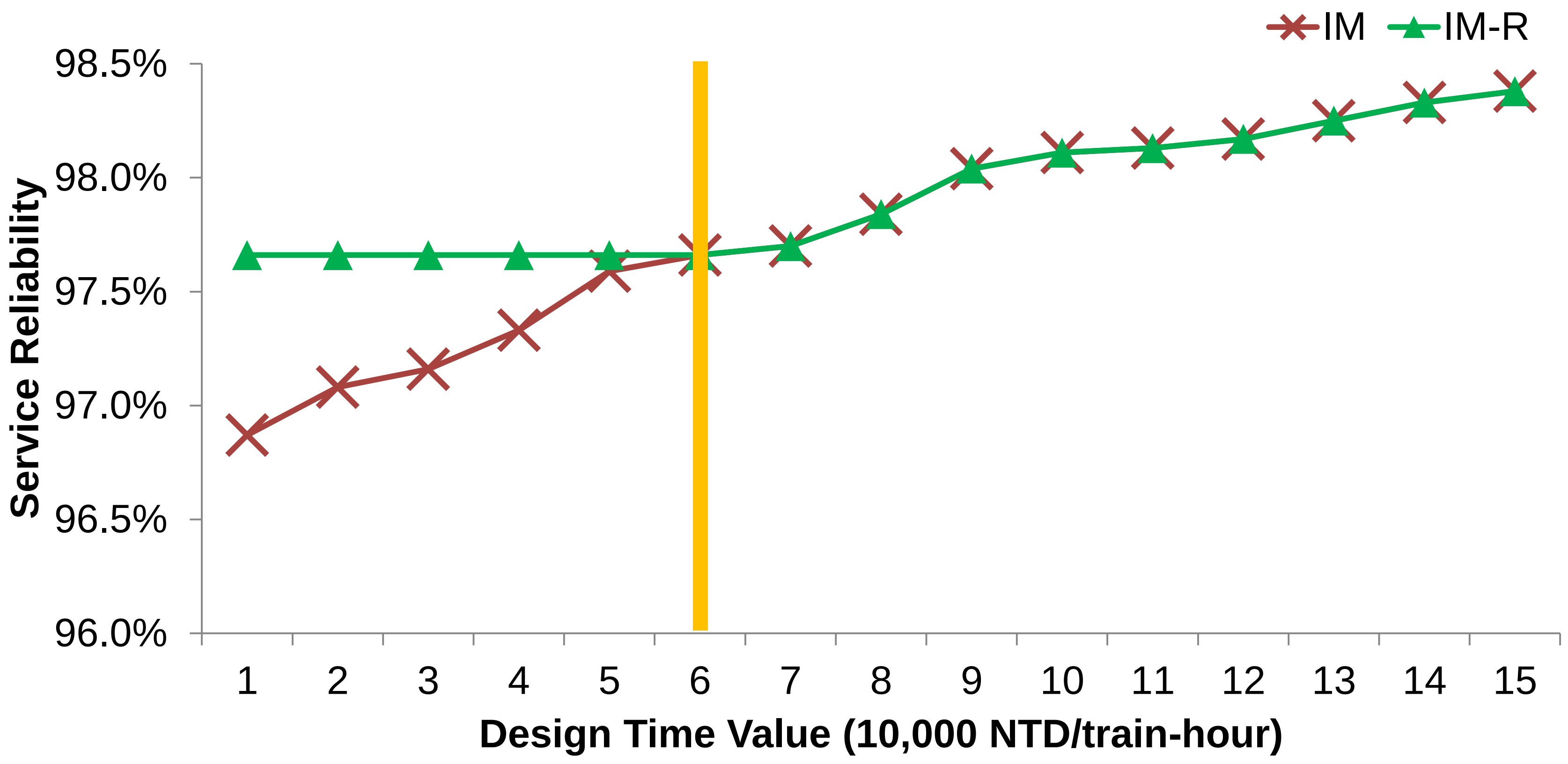


IM-B will be constrained by LCC budget

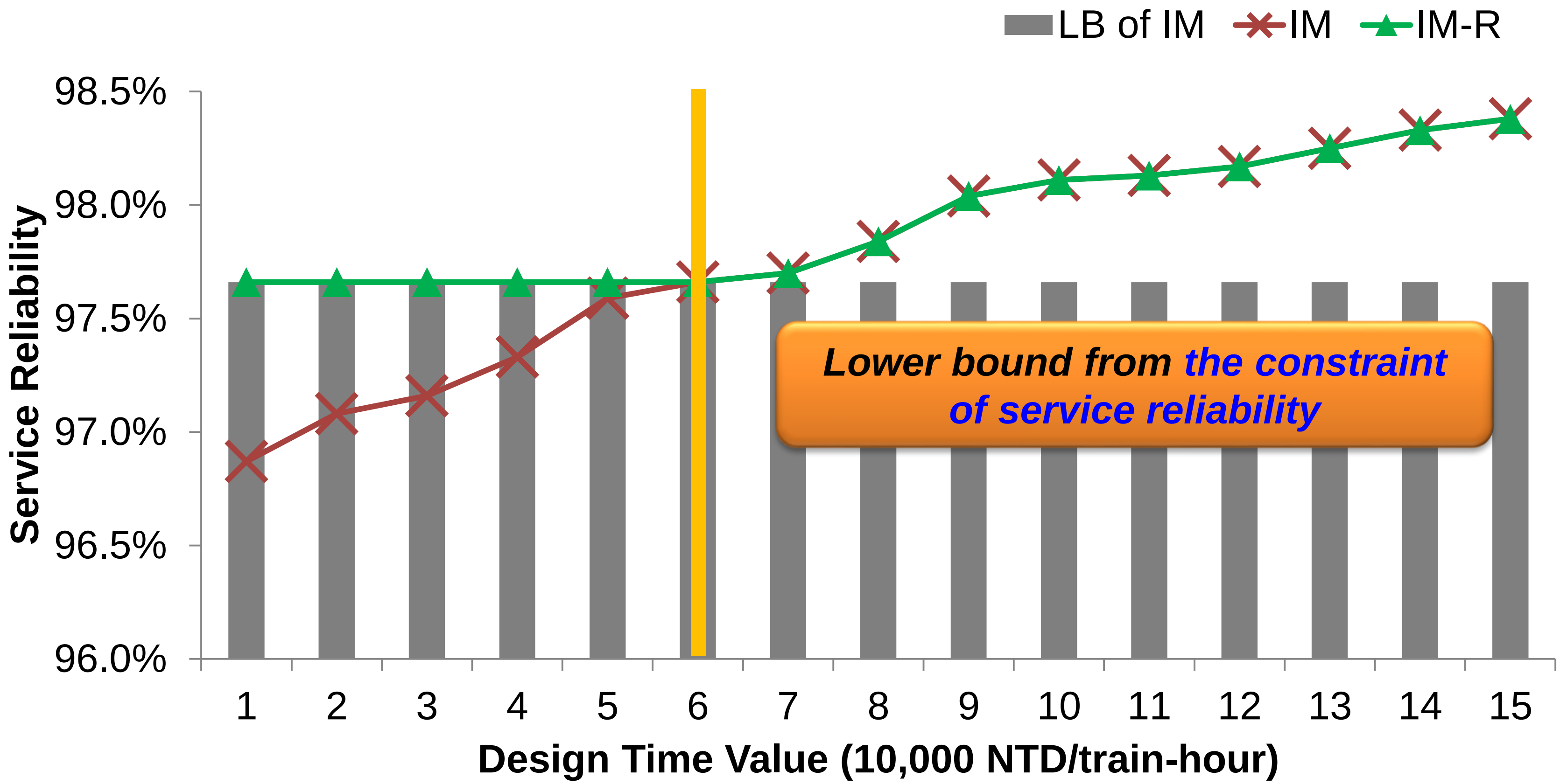


IM-B will be constrained by 100 billion NTD LCC budget when design time value is more than about 110,000 NTD/train-hour

IM-R will be constrained by design service reliability

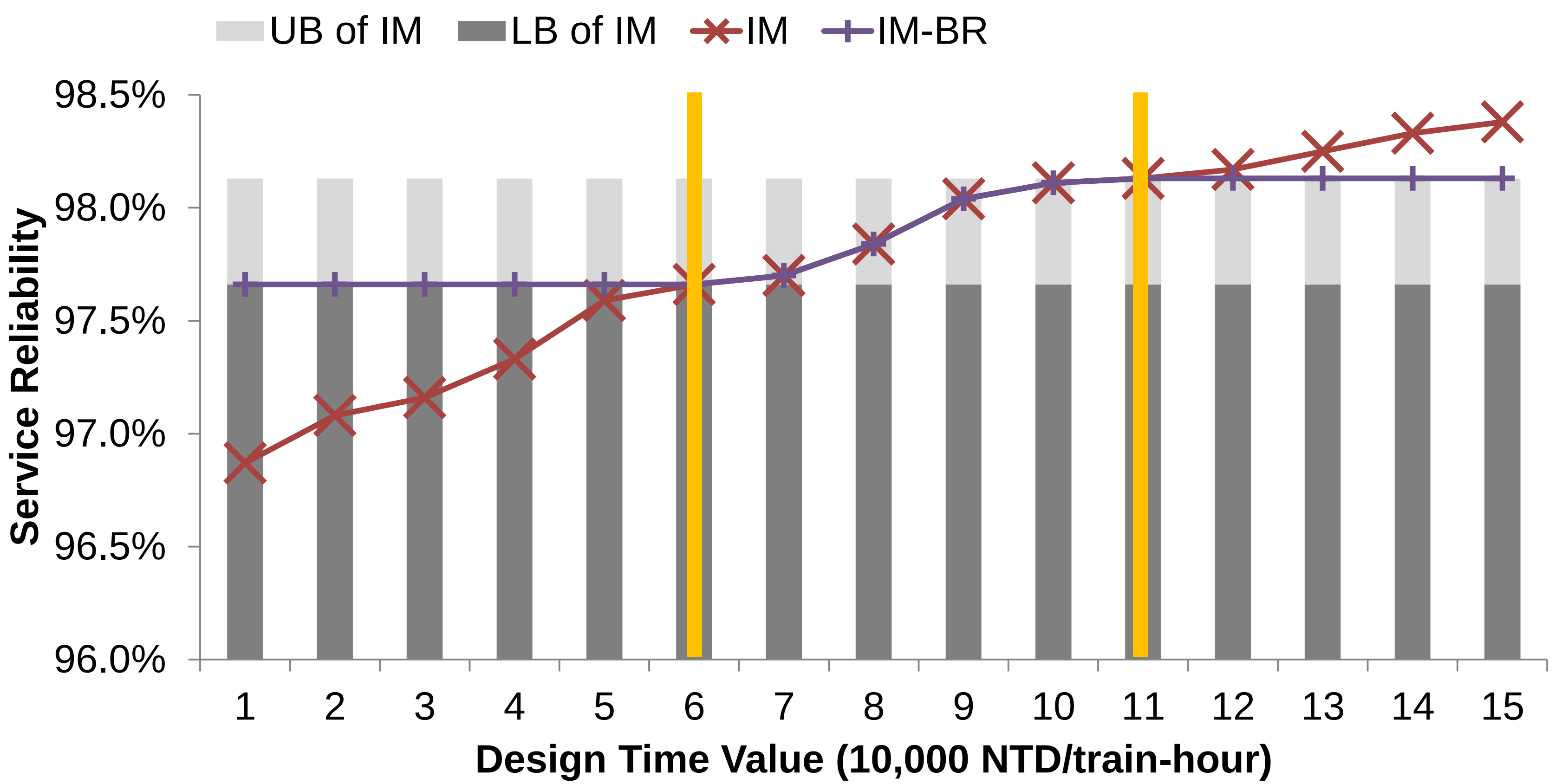


IM-R will be constrained by design service reliability



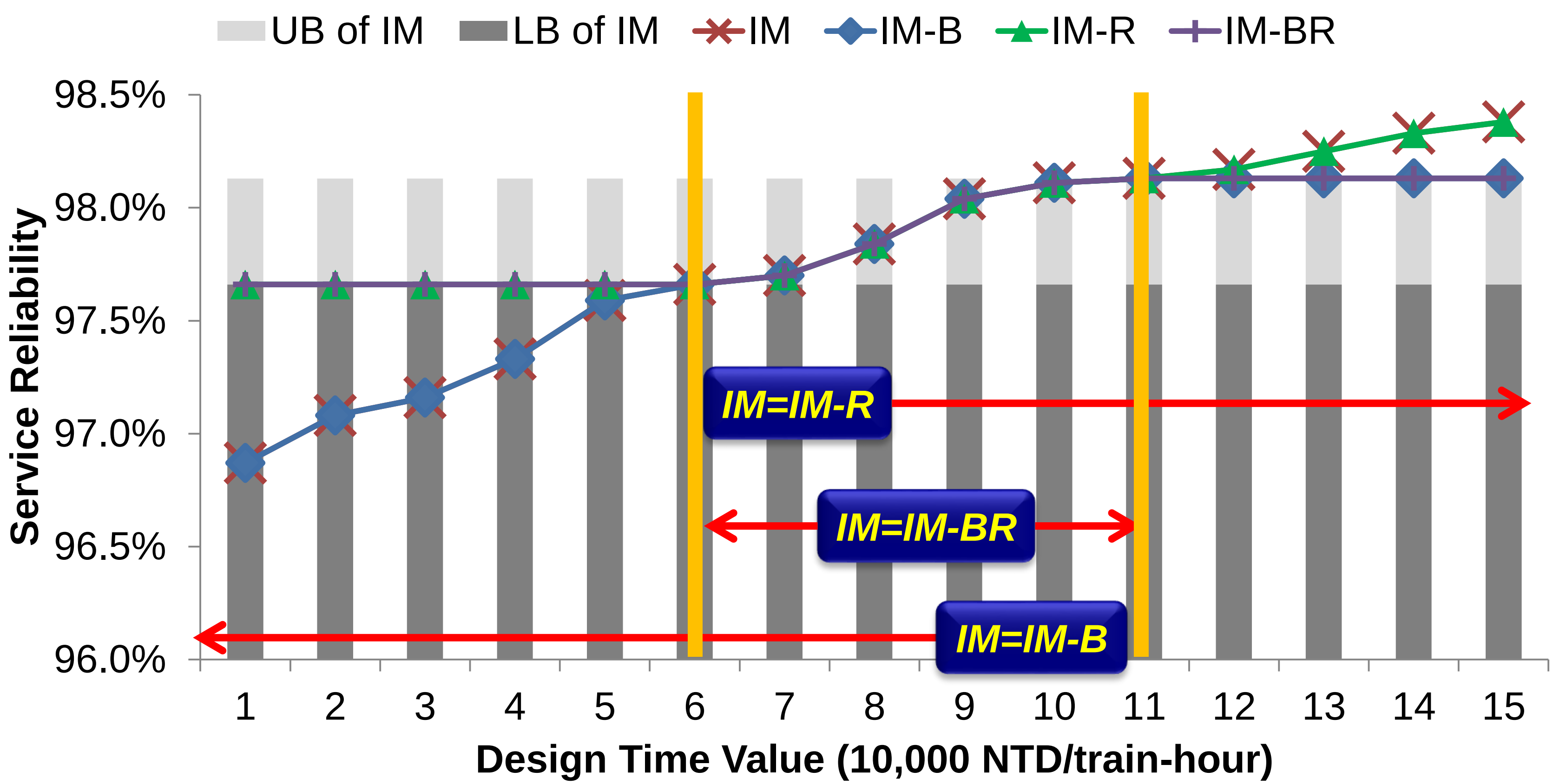
IM-R will be constrained by 97.7% service reliability when design time value is less than about 60,000 NTD/train-hour

IM-BR will be constrained by both LCC and service reliability



IM-BR is constrained by budget and service reliability; it has the same trend as IM in terms of time value between 60,000~110,000 NTD/train-hour

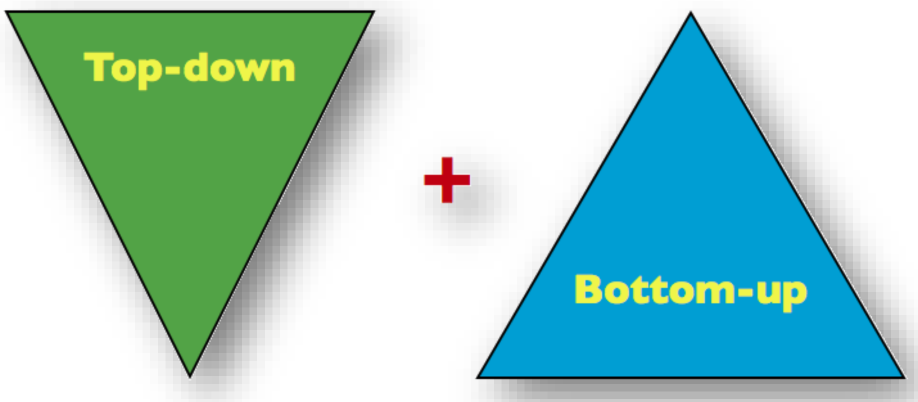
IM-BR will be constrained by both LCC and service reliability



This integrated model framework is flexible according to planners' need in rail system design

Conclusion

- This research develops an **optimization process** to assist decision makers in optimally allocating service reliability, system reliability, and LCC
- It is essential to incorporate **service reliability** in rail system design



The proposed tool can help railways maximize their return on investment and provide reliable service to passengers

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Thank You!

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